

Cycles and paths in line graphs of 3-hypergraphs

Zdeněk Ryjáček

(joint work with Binlong Li, Kenta Ozeki, and Petr Vrána)

In 1986, Thomassen conjectured that every 4-connected line graph is hamiltonian. The conjecture is still wide open, and, as a possible approach to it, many statements that are equivalent or related to it have been studied. We extend the statement to the class of line graphs of 3-hypergraphs, and generalize it to Tutte cycles and paths (note that a line graph of a 3-hypergraph is $K_{1,4}$ -free but can contain induced claws $K_{1,3}$, and that a Tutte cycle/path is a cycle/path such that any component of its complement has at most three vertices of attachment). Among others, we formulate the following conjectures:

- (i) every 2-connected line graph of a 3-hypergraph has a Tutte maximal cycle containing any two prescribed vertices,
- (ii) every 3-connected line graph of a 3-hypergraph has a Tutte maximal cycle containing any three prescribed vertices,
- (iii) every connected line graph of a 3-hypergraph has a Tutte maximal (a, b) -path for any two vertices a, b ,
- (iv) every 4-connected line graph of a 3-hypergraph is Hamilton-connected, and we show that all these (seemingly much stronger) statements are equivalent with Thomassen's conjecture.