Upper bounds on the orders of $(\boldsymbol{k}, \boldsymbol{g})$-graphs derived from conditions on their spectra

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(joint work with Robert Jajcay)

A (k, g)-graph is a k-regular graph of girth g. The Cage Problem has been extensively studied since the 1960's when Erdős and Sachs proved the existence of infinitely many (k, g)-graphs for any pair (k, g). They also found a general constructive bound for any pair (k, g) which was later improved by Sauer. In both cases, it was proven that starting of the value of the bound for a given (k, g)-value, there exist (k, g)-graphs of all admissible higher orders. Recently, we have developed recursive techniques for constructing families of (k, g)-graphs of increasing order. We have also introduced new properties required of (k, g)-graphs which are sufficient to guarantee the possibility of adding vertices to a given (k, g)-graph, and thereby obtaining a bigger (k, g)-graph. From these conditions we derive upper bounds which guarantee the existence of (k, g)-graphs of all admissible higher orders. We compare these bounds to the constructive upper bounds obtained by Sauer, which still constitute the best general upper bounds for (k, g)-graphs.