Planar graphs have bounded queue-number

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We show that planar graphs have bounded queue-number, thus proving a conjecture of Heath, Leighton and Rosenberg from 1992. The key to the proof is a new structural tool called layered partitions, and the result that every planar graph has a vertex-partition and a layering, such that each part has a bounded number of vertices in each layer, and the quotient graph has bounded treewidth. This result generalises for graphs of bounded Euler genus. Moreover, we prove that every graph in a minor-closed class has such a layered partition if and only if the class excludes some apex graph. Building on this work and using the graph minor structure theorem, we prove that every proper minor-closed class of graphs has bounded queue-number. Layered partitions have strong connections to other topics, including the following two examples. First, they can be interpreted in terms of strong products. We show that every planar graph is a subgraph of the strong product of a path with some graph of bounded treewidth. Similar statements hold for all proper minor-closed classes. Second, we give a simple proof of the result by DeVos et al. (2004) that graphs in a proper minor-closed class have low treewidth colourings. The talk is based on results from [1].

References

[1] V. Dujmović, G. Joret, P. Micek, P. Morin, T. Ueckerdt, D. Wood, Planar graphs have bounded queue-number, arXiv:1904.04791 (2019).