

On practical aspects of snark generation

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(joint work with Jozef Rajník)

Bridgeless cubic graphs that do not admit a 3-edge-colouring, called snarks, form a fairly important class of graphs, especially if one is interested in problems related to flows and cycle covers. In the ongoing hunt for snarks, it has been discovered that some are just simple modifications of others. It has also transpired that snarks with larger girth, and especially those with larger cyclic connectivity, are the rarest and thus the most highly regarded. In particular, it has been conjectured that there are no cyclically 7-connected snarks, and apart from the infinite family of Isaacs snarks, the smallest known cyclically 6-connected snark has 118 vertices and there are no such snarks with up to 38 vertices. The only known construction method that does not decrease cyclic connectivity below 6 is superposition which tends to produce big graphs (typically with hundreds of vertices). Our focus will be on cyclically 5-connected snarks with girth at least 6 (which are “the next best thing” since cyclic 6-connectivity implies girth at least 6) which are as small as possible. In particular, we construct such a snark on 52 vertices and prove that none exists on 40 vertices. The proof is based on an exhaustive computer search that starts with generating certain lists of so-called trivial snarks (e.g., 3-connected on up to 30 vertices). For this purpose, we employed Slovak Infrastructure for High Performance Computing which is not widely known in the Slovak combinatorial community, so a secondary aim of this talk is to discuss some practical aspects of larger-scale computations.