Short cycles in plane graphs

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The classic consequence of Euler polyhedral formula states that each polyhedral graph (or, more generally, each plane graph of minimum degree at least 3) contains a face bounded by a cycle of length 3, 4 or 5. Based on this result, it is possible to consider several directions of research of cycles of fixed (short) lengths in plane graphs. Nowadays, a well explored topic (see the more general survey [1]) covers the existence of fixed short cycles (or short cycles of length from certain range) with additional (so called *lightness*) property that all vertices have (relatively) low degrees. We present the overview of results on light cycles and light sets of cycles in various families of plane graphs as well as results dealing with the question how far is a particular cycle from being light [2]. Another approach studies just presence (or absence) of certain cycles in plane graphs; despite the relaxation of the lightness property, the proofs of positive results on existence of particular short cycles are not much easier, as shown by recent results from [3, 4].

References

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[4] T. Madaras, M. Tamášová, Minimal unavoidable sets of cycles in plane graphs with restricted minimum degree and edge weight, Australas. J. Combin. 74 (2019) 240–252.