3-choosability of planar graphs with maximum degree 4

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(joint work with François Dross, Borut Lužar, and Roman Soták)

Deciding whether a planar graph (even of maximum degree 4) is 3-colorable is NP-complete. Therefore, it is a natural question to determine which classes of graphs are 3-colorable. Heawood proved that plane triangulation is 3-colorable if and only if all its vertices have even degrees. On the other hand, a well-known result by Grötzsch shows that if there are no cycles of length 3 in a planar graph, then it is 3-colorable. Allowing some triangles in a graph, but still retaining 3-colorability yielded two intriguing conjectures. Havel conjectured that a 3-colorable planar graph may contain many triangles as long as they are sufficiently far apart. This conjecture was recently proved by Dvořák, Král', and Thomas. The second conjecture is due to Steinberg and it allows arbitrary many triangles but it forbids short cycles. Namely, Steinberg conjectured that every planar graph without cycles of length 4 and 5 is 3-colorable. The conjecture was disproved by Cohen-Addad et al. In this talk, we prove that every planar graph obtained as a subgraph of the medial graph of any bipartite plane graph is not only 3-colorable, but also 3-choosable. These graphs are allowed to have close triangles (even incident), and have no short cycles forbidden, hence representing an entirely different class than the graphs inferred by the above mentioned conjectures.