Short cycle covers of graphs with minimal degree 3

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(joint work with Anna Kompišová)

Let G be a bridgeless graph. A *cycle cover* of G is a set of circuits containing every edge of G. The *length* of a cycle cover is the sum of lengths of its circuits. Short cycle conjecture asserts that each bridgeless graph G has a cycle cover of length at most $1.4 \cdot |E(G)|$. For general graphs the best bound $5/3 \cdot |E(G)| \approx 1.666 \cdot |E(G)|$ [1, 2] was proven already in the 80's. In this talk we restrict ourselfs to graphs without vertices of degree two. Kaiser et al. [4] proved that such graphs have cycle cover of length at most $\approx 1.630 \cdot |E(G)|$ using a combination of methods from [1] and [2] which is also a principal approach we rely on. These ideas were refined by Fan [3] who proved the bound $\approx 1.615 \cdot |E(G)|$. Both results were obtained for loopless graphs but can be generalized to graphs with loops [5]. In this talk we sketch how to show that every bridgeless graph without vertices of degree two has a cycle cover of length less than $1.589 \cdot |E(G)|$.

References

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