

# Extremal problems concerning tournaments

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A tournament is an orientation of a complete graph. We provide a brief overview of various extremal problems related to tournaments, particularly those concerning cycles. We particularly focus on the conjecture of Linial and Morgenstern that, among all tournaments with a given density  $d$  of cycles of length three, the density of cycles of length four is minimized by a random blow-up of a transitive tournament with all but one parts of equal sizes, i.e., a tournament with the structure similar to graphs appearing in the Erdős-Rademacher problem on triangles in graphs with a given edge density. We prove the conjecture of Linial and Morgenstern for  $d \geq 1/36$  using methods from spectral graph theory. We also demonstrate that the structure of extremal examples is more complex than expected and give its full description for  $d \geq 1/16$ .