Classification of edge-transitive maps and group actions on orientable surfaces

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(joint work with Roman Nedela and Mária Skyvová)

A map $\mathbf{M}$ is edge-transitive if its group of automorphisms, $\mathrm{Aut}\,\mathbf{M}$, acts transitively on the edges of the underlying graph of $\mathbf{M}$. The group of orientation-preserving automorphisms, is a subgroup $\mathrm{Aut}^+\,\mathbf{M}$ of index at most two in $\mathrm{Aut}\,\mathbf{M}$. It follows that the quotient map of an edge-transitive map, $\bar{\mathbf{M}} = \mathbf{M}/\mathrm{Aut}^+\,\mathbf{M}$, is a map on an (quotient) orbifold with at most two edges. There are exactly 8 quotient maps sitting on orbifolds with at most 4 singular points, seven are spherical and one is toroidal. We give the classification of edge-transitive maps on orientable surfaces of genera $1 < g \leq 101$. More precisely, for each of the 8 families, the classification reduces to the problem of determining normal subgroups of bounded index in the associated groups of automorphisms of the universal cover of maps. The modified well-known technique of voltage assignments and regular covers [2] to reconstruct the edge-transitive maps on orientable surfaces of given genera. Compared to the methods used in [3, 4] we control the genus of the underlying surface by choosing a proper g-admissible orbifold. Moreover, we examine action of reflections in quotient maps to establish the relationship between our classification, the one given by Širáň et al. [4] and the classical result by Graver and Watkins [1].

References

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