An upper bound for large graphs of given degree and diameter two arising from Abelian lifts of complete bipartite multigraphs

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The degree-diameter problem is to determine the largest number $n_{d,k}$ of vertices in a graph of maximum degree d and diameter k. If the diameter is 2, then $n_{d,2} \leq d^2+1$. Some constructions giving large graphs of given degree and diameter two rely on lifting small graphs; prominent example are the McKay-Miller-Širáň graphs [1] of order $\frac{8}{9}(d+\frac{1}{2})^2$ for all degrees $d = \frac{3q-1}{2}$, where q is a prime power congruent to 1 (mod 4), obtained as Abelian lifts of suitable complete bipartite graphs. The purpose of this paper is to show that there is no better bound on the order of Abelian lifts of complete bipartite multigraphs of diameter 2 than the order of the McKay-Miller-Širáň graphs. That is, we derive the upper bound $\frac{8}{9}(d+\frac{1}{2})^2$ for the order of Abelian lift of any complete bipartite multigraph, regular of degree $d \geq 10$, with given edge multiplicity and with the same number of loops and semi-edges at each vertex.

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References

[1] B.D. McKay, M. Miller, J. Širáň, A note on large graphs of diameter two and given maximum degree, J. Combin. Theory Ser. B 74 (1998) 110–118.