Axes-parallel unit disk graph recognition is NP-hard

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Unit disk graphs are the intersection graphs of unit radius disks in the Euclidean plane. Recognizing a unit disk graph is an important geometric problem, and has many application areas. In general, this problem is shown to be NP-hard [3], and even $\exists\mathbb{R}$-complete [7]. In some applications, the objects that correspond to unit disks, have predefined (geometrical) structures to be placed on. Hence, many researchers attacked this problem by restricting the domain of the disk centers. One example to such applications is wireless sensor networks, where each disk corresponds to a wireless sensor node, and a pair of intersecting disks corresponds to a pair of sensors being able to communicate with each other. It is usually assumed that the nodes have identical sensing ranges, and thus unit disk graph model is used to model problems concerning wireless sensor networks. In a wireless sensor network, the sensor nodes are deployed on bounded areas [2, 4]. Thus, it becomes more interesting to observe the behavior of the unit disk graph recognition problem when the domain is restricted [1, 6]. In this paper, we also attack the unit disk recognition problem on a restricted domain, by assuming a scenario where the wireless sensor nodes are deployed on the corridors of a building. Based on this scenario, we impose a geometric constraint such that the unit disks must be centered onto given straight lines. We show that deciding whether there exists a realization of a given graph as unit disks on straight lines is NP-hard, even if the given lines are parallel to either x-axis or y-axis. We use a reduction from Not-all-equal 3-satisfiability problem, and utilize the logic engine introduced by Fekete et al. [5]. Moreover, we remark that if the straight lines are not given, then the problem becomes $\exists\mathbb{R}$-complete.

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